Magnet Basics

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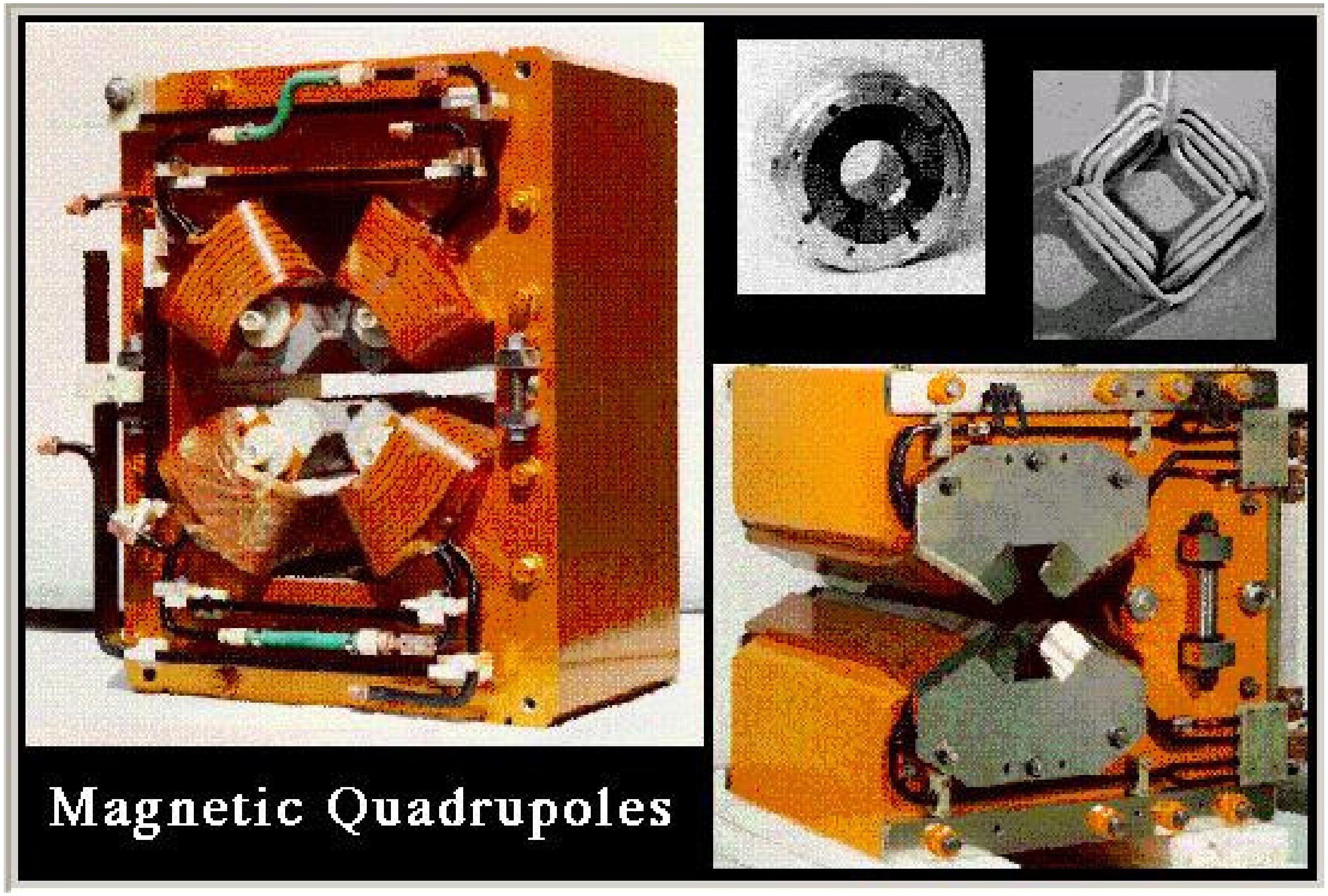
Magnets: Introduction

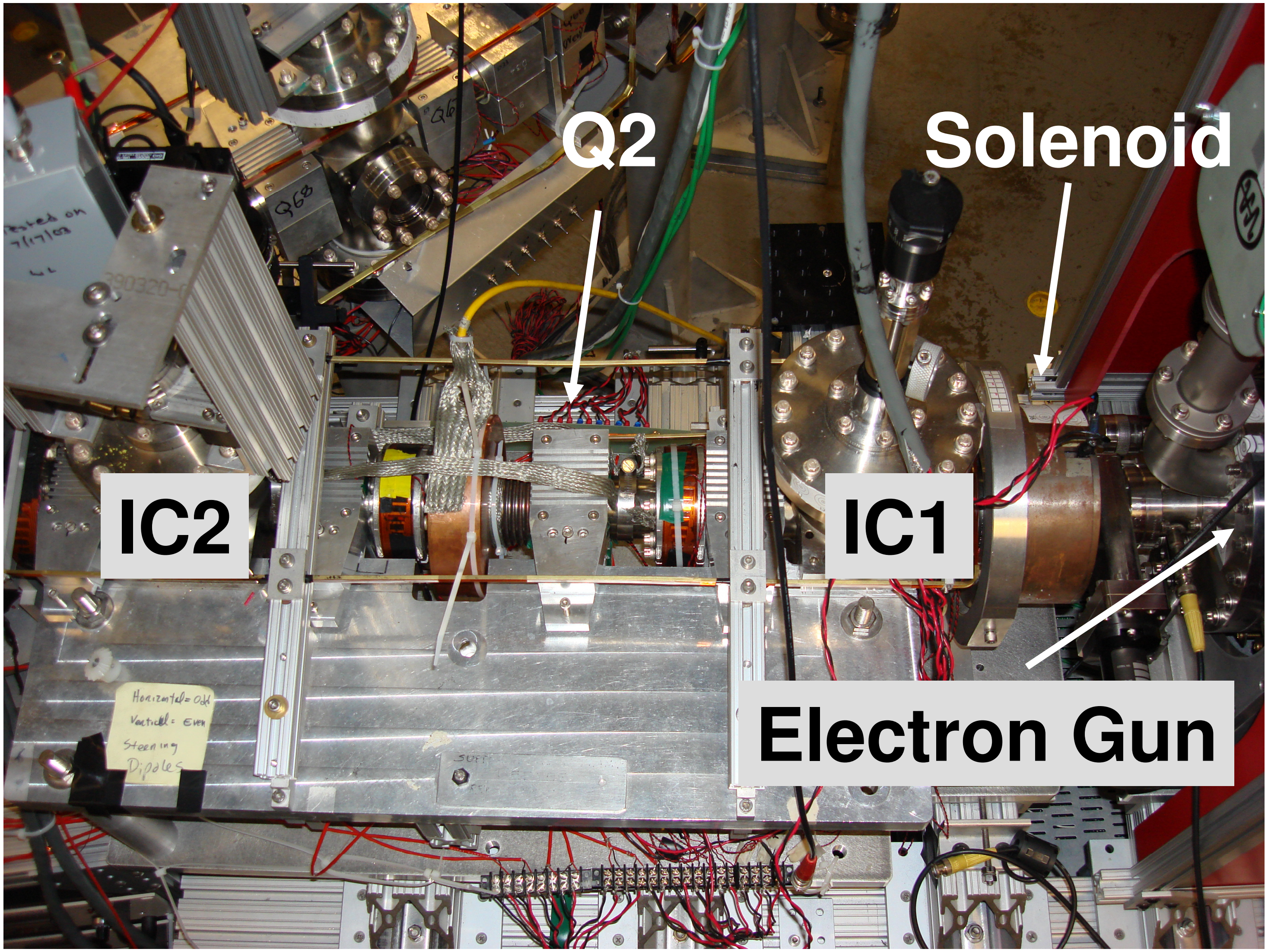
- Magnets are key components of all accelerators
- Magnet modeling has several stages:
  1. Simple hardedge models for optics design (energy and type of charged particle are main considerations)
  2. Computer calculations
  3. Mechanical/electrical design and construction
  4. Magnet measurement:

     field/gradient profile and/or multipole measurement
  5. Beam testing
- Item 1 requires a magnet strength per amp or volt, and an effective length. Items 2 and 4 yield actual values.
- Measurement devices: gaussmeters (e.g. Hall-effect), rotating coils, taut wire techniques, etc.
- Computer codes: OPERA3D/TOSCA, MERMAID, AMPERE, MAG-Li

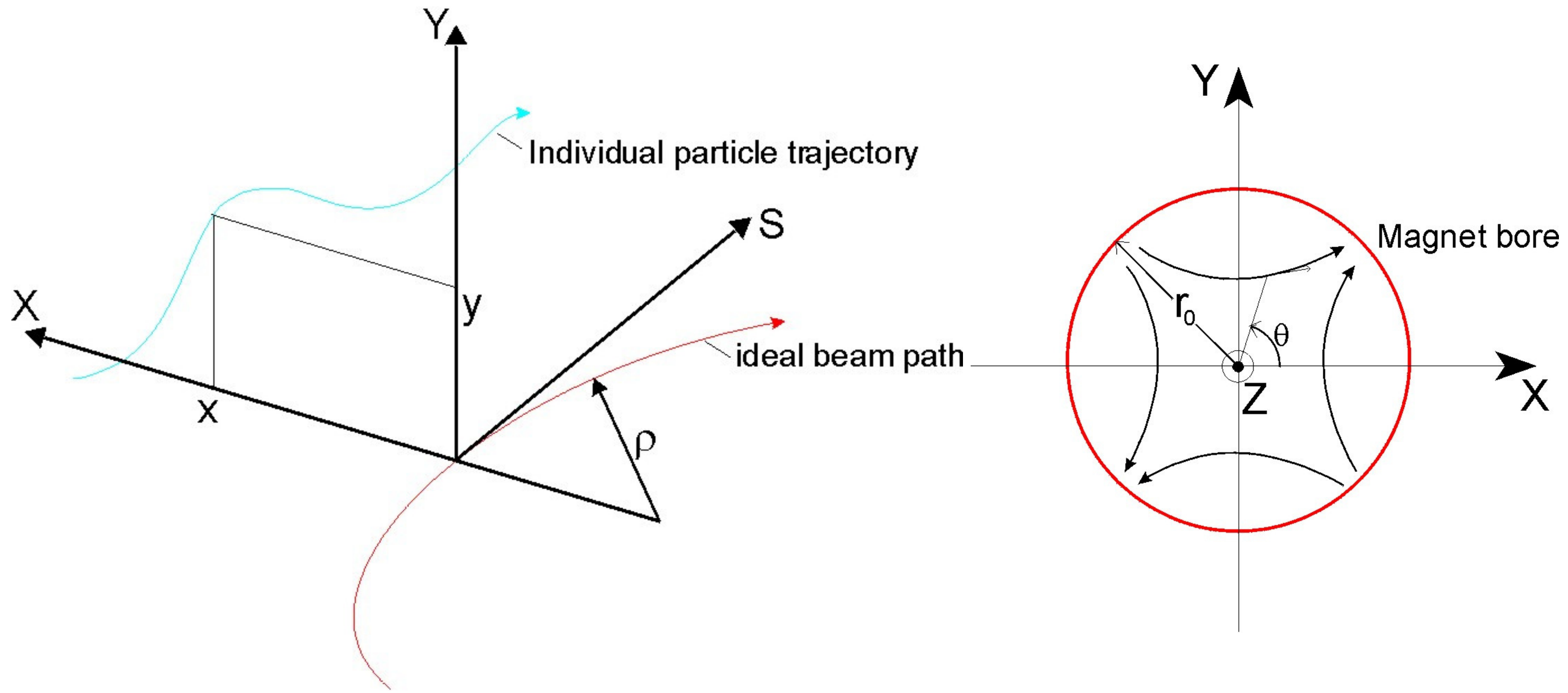
Magnets: Introduction



Magnets in UMER: Matching Section



Coordinate Systems and Notation



Bending dipoles define **reference trajectory.**

$\gamma m v^2/\rho = qvB \rightarrow B\rho = p/q$: Magnetic Rigidity

Defined as:

$$B\rho = \frac{p}{q}, \qquad p = \gamma m \beta c$$

For relativistic e⁻:

$$B\rho = \frac{p}{e} \cong 0.3\,pc$$

Tesla m GeV/c

Bending:
dθ = ds/ρ(s), so

$$\theta_2 - \theta_1 = \int_{s_1}^{s_2} \frac{ds}{\rho(s)} \cong \frac{3.0}{pc} \int_{s_1}^{s_2} B(s)\,ds$$

Focusing:

$$x''(z) + \kappa_{0x} x(z) = 0, \quad y''(z) + \kappa_{0y} y(z) = 0.$$

- Quadrupole: $\kappa_{0x} = -\kappa_{0y} = \dfrac{g_0}{(B\rho)}$
- Solenoid: $\kappa_0 = \dfrac{B_z^{\,2}}{4(B\rho)^2}$ $\qquad r''(z) + \kappa_0 r(z) = 0$

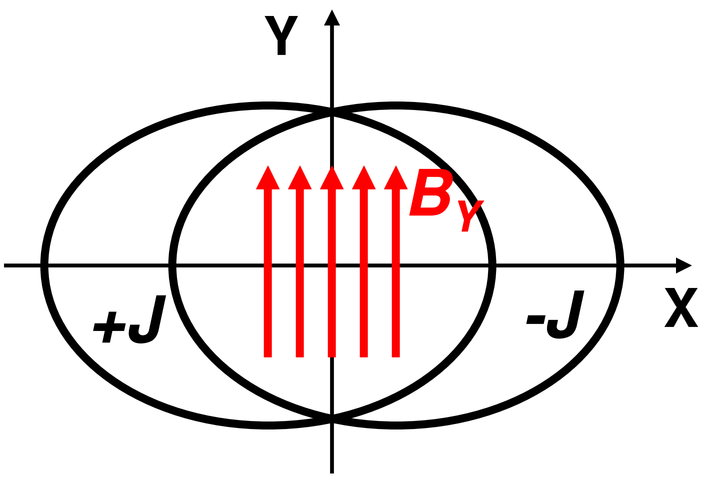
Magnets: Introduction

Separated Function vs. Combined Function

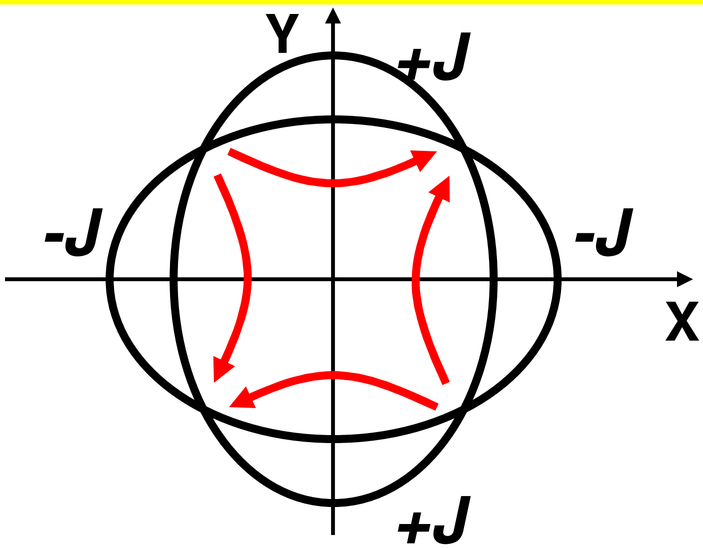
Dipoles, **Quadrupoles**, Sextupoles, Octupoles

Electrostatic vs. **Magnetostatic**

Displaced, overlapping (& infinite) solid elliptical cylinders carrying uniform current density generate **pure fields**:



Pure **Dipole**

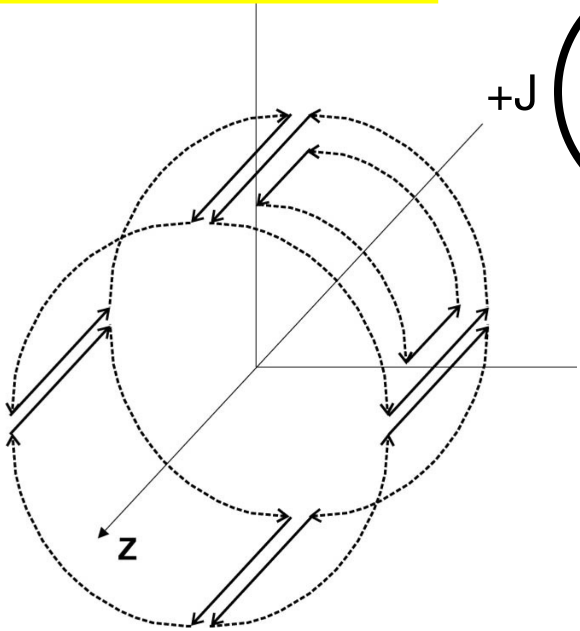


Pure **Quadrupole**

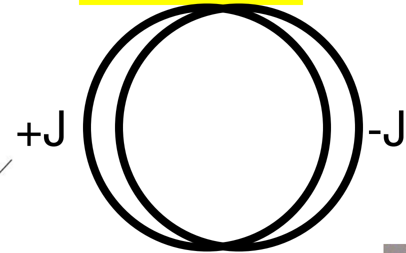
UMER PC Dipole and Quadrupole*

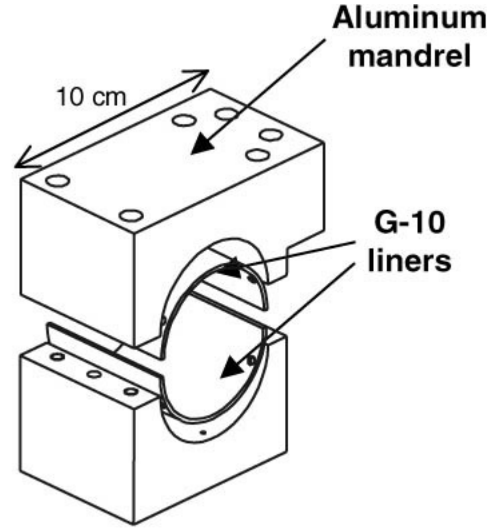
On a circular cylindrical surface, we want: $\int K_z dz \propto cosn\theta$

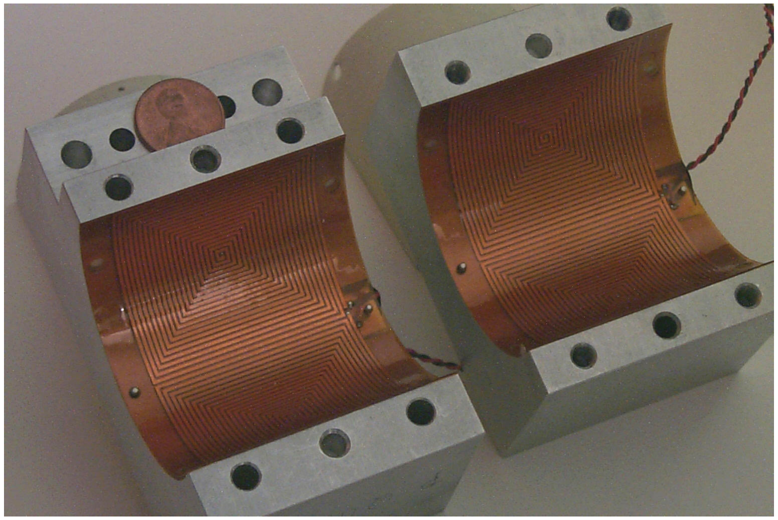
n=order of multipole.



Recall…







UMER PC quadrupole

Only conductors parallel to *z*-axis contribute to integrated *B*-field.

*W.W. Zhang, et al, Phys, Rev. ST Accel. Beams, 3, 122401 (2000).

Multipole Expansion

2D Multipole Expansion:

$$B(x,y) = B_y + iB_x = \sum_{n=1} (b_n + ia_n) \left(\frac{x+iy}{r_0} \right)^{n-1},$$

$$r = \sqrt{x^2+y^2} < r_0, \qquad (1)$$

b_n = Normal Component,
a_n = Skew Component
r_0 = Aperture Radius

$$B_r(r,\theta) = \sum_{n=1} \left(\frac{r}{r_0} \right)^{n-1} [b_n Sin(n\theta) + a_n Cos(n\theta)],$$

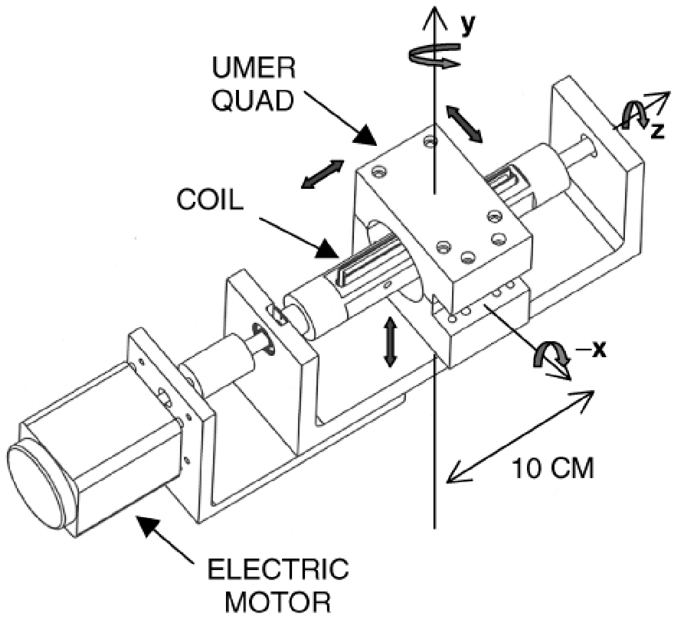
$$B_\theta(r,\theta) = \sum_{n=1} \left(\frac{r}{r_0} \right)^{n-1} [b_n Cos(n\theta) - a_n Sin(n\theta)]. \qquad (2)$$

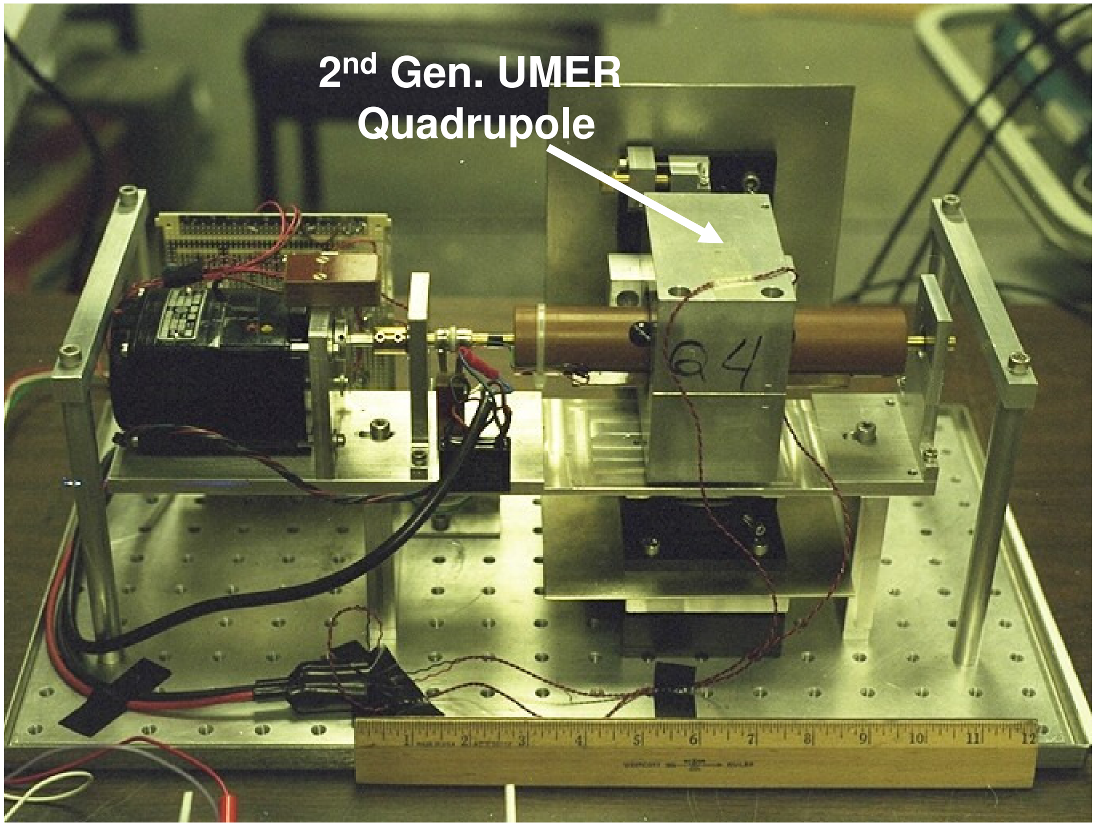
3D Multipole Expansion:
$B \rightarrow B^{Int}$

From symmetry, a magnet with **quadrupole symmetry** has only multipoles of the form $n = 4k+2$ (k=0,1,2, …), i.e. **quadrupole** (n=2), **duodecapole** (n=6), 10-pole (n=10), etc.

WE WANT SMALL UNDESIRED MULTIPOLES:
typically less than 1 part in 10^4

UMER Rotating Coil*

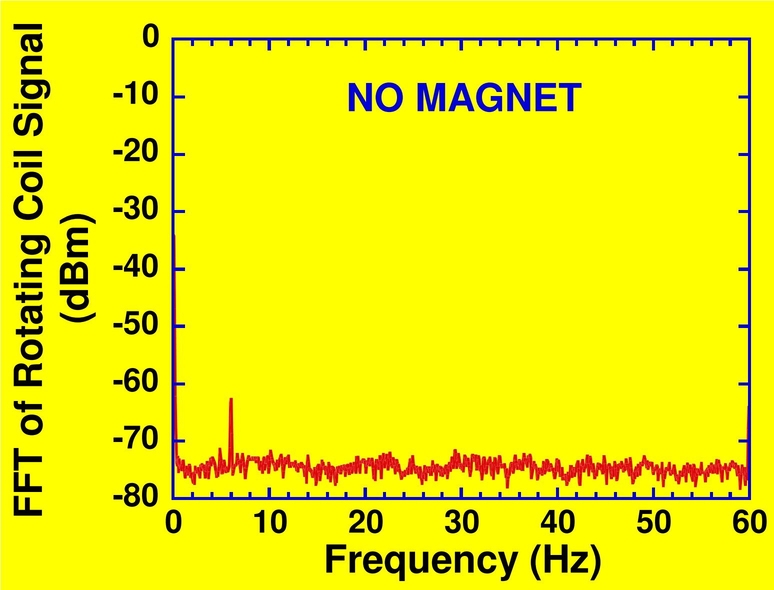


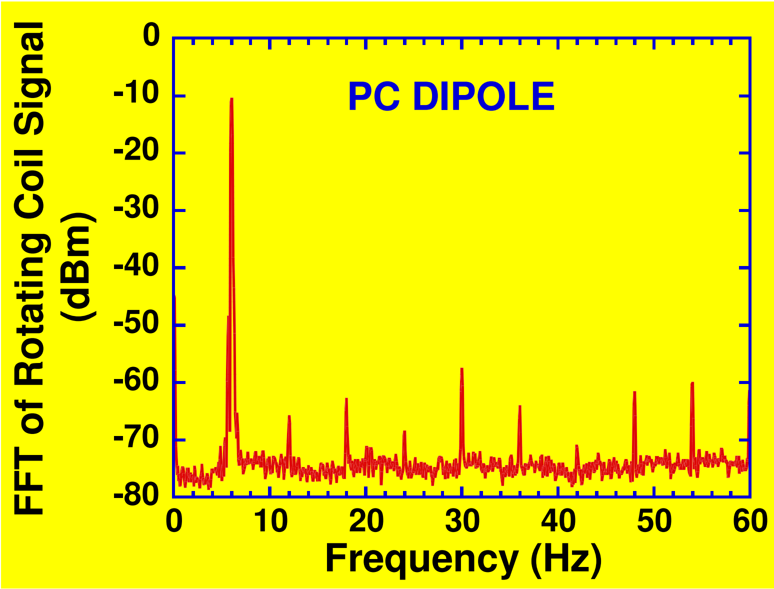


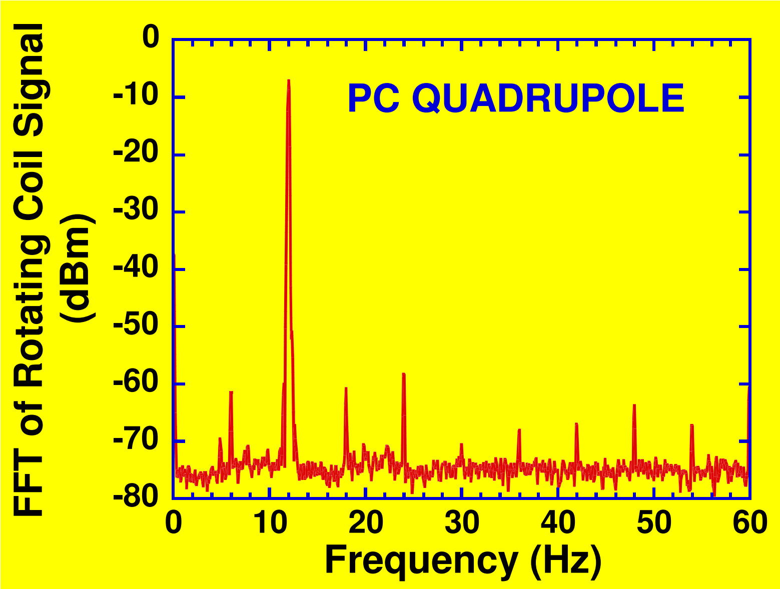
The coil contains ~3000 turns of very fine wire. The whole of the rotating coil apparatus is normally enclosed in mu-metal box.

*W.W. Zhang, et al, Phys, Rev. ST Accel. Beams, 3, 122401 (2000).

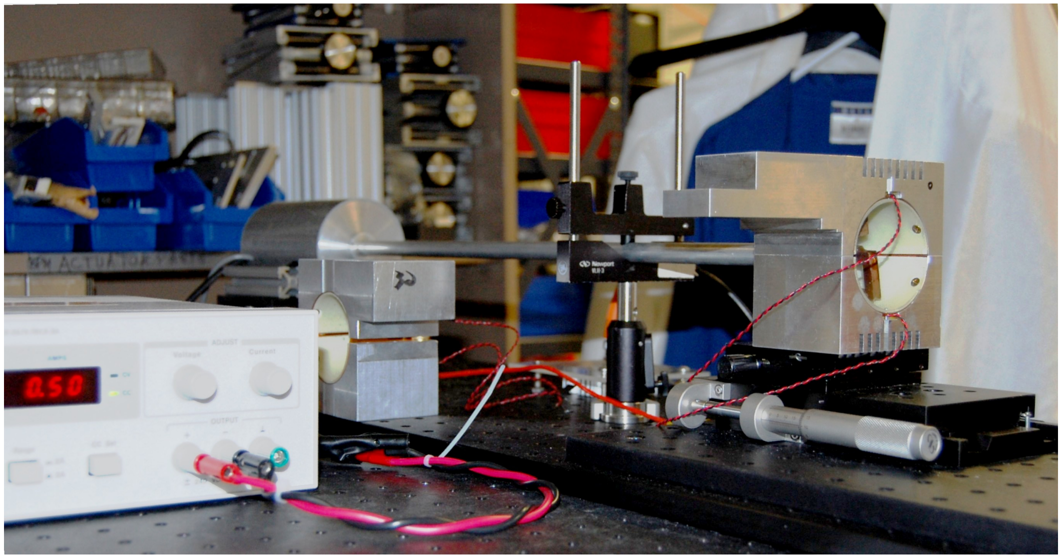
FFT of Rotating Coil Signal

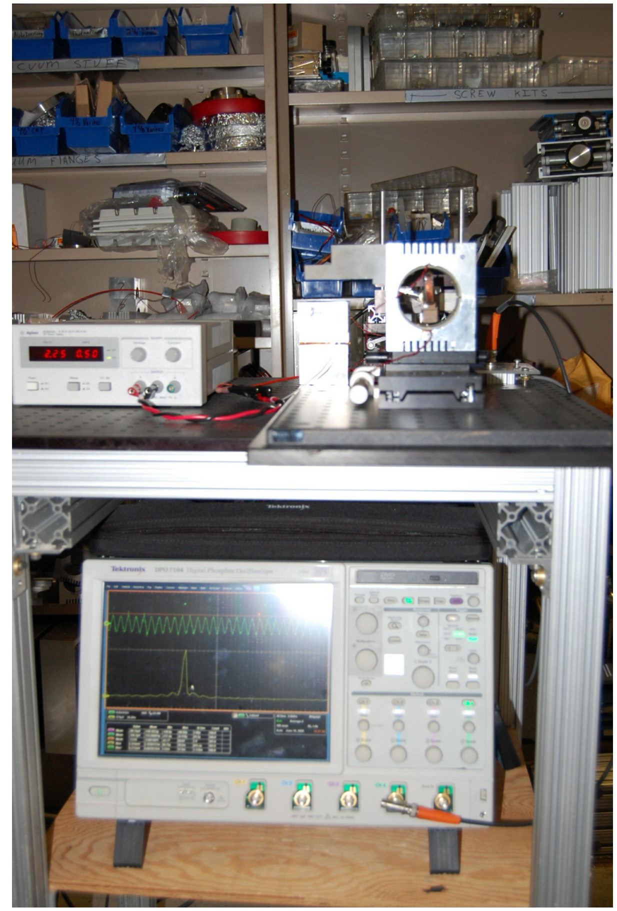




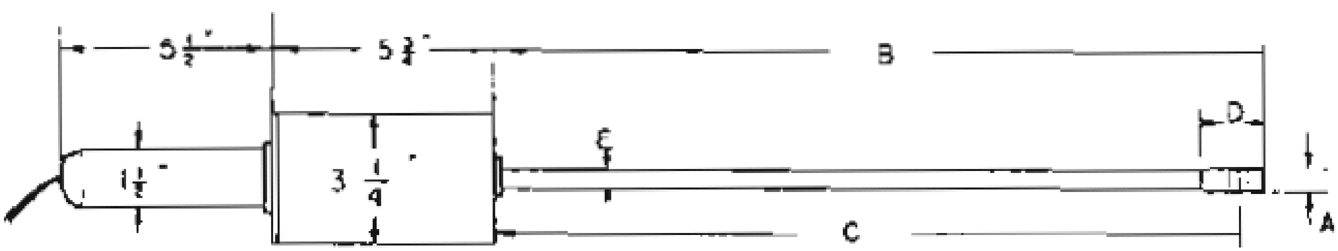


UMER Simple Rotating Coil

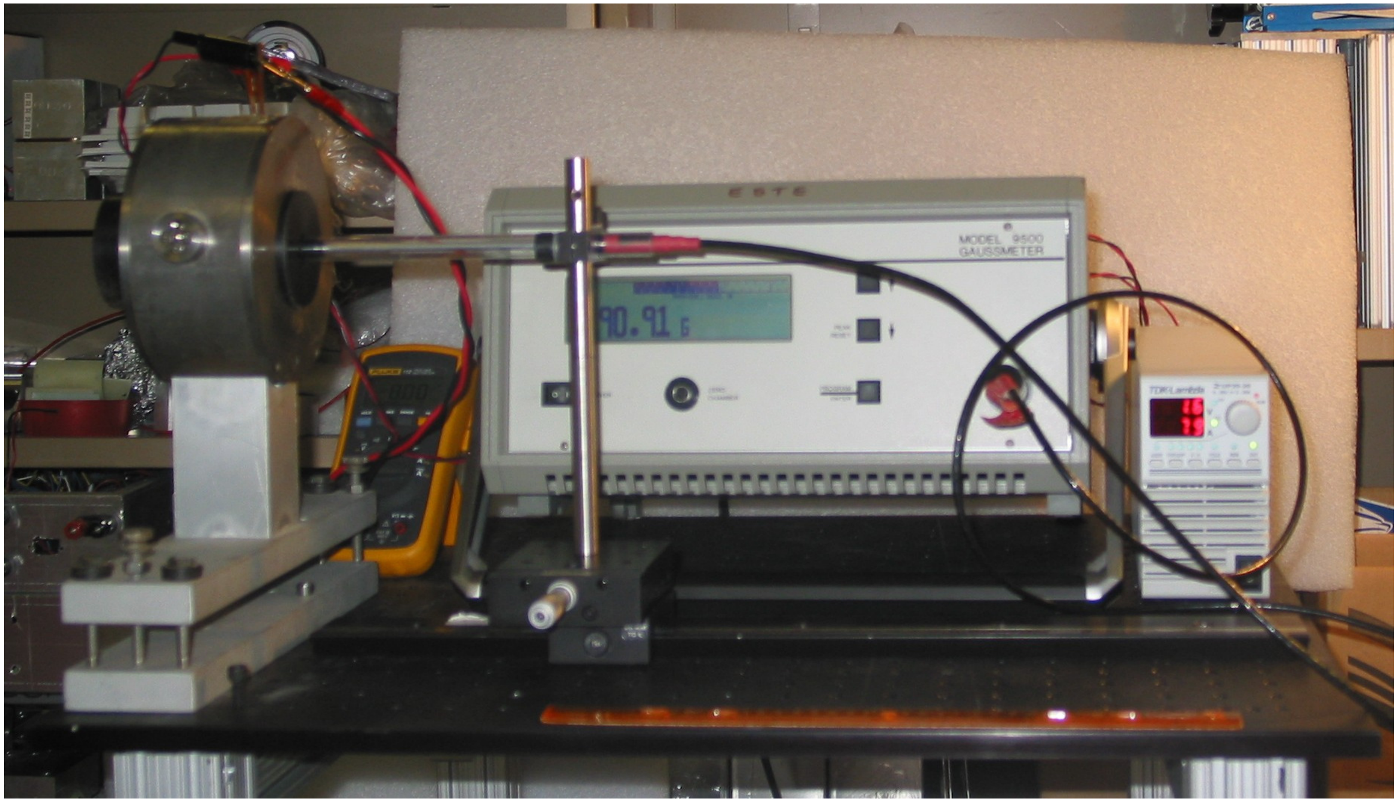




Rawson-Lush rotating coil gaussmeter
Model 780:
Tip Diameter. **A**: 6.35 mm
Probe Length **B**: 50.0 cm
Length to coil center **C**: 48.9 cm
Tube Diameter **E**: 6.35 mm



Short Solenoid: Axial Field Profile Measurement

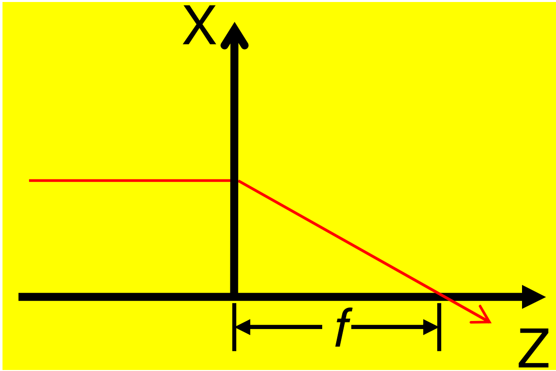


For **axially symmetric *B*-fields**, components B_z, B_r can be found at all (z, r) from knowledge of $B_z(r=0,z)=B(z)$, **i.e. the on-axis field profile:**

$$B_r(r,z) = -\frac{r}{2}\frac{\partial B}{\partial z} + \frac{r^3}{16}\frac{\partial^3 B}{\partial z^3} - \cdots,$$

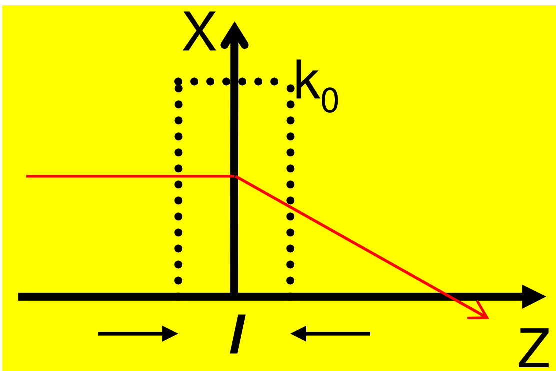
$$B_z(r,z) = B - \frac{r^2}{4}\frac{\partial^2 B}{\partial z^2} + \frac{r^4}{64}\frac{\partial^4 B}{\partial z^4} - \cdots$$

Review: Modeling of Lenses



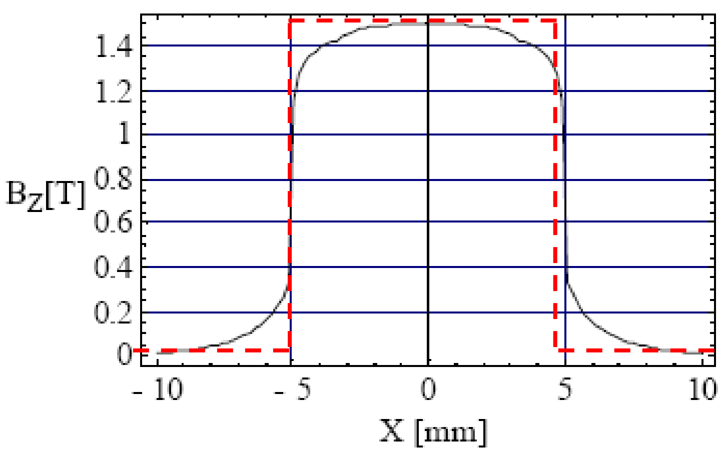
"Point" Lens:

$$x''(z) + \frac{\delta(z)}{f} x(z) = 0$$



Thin Hard-Edge (f>>l):

$$x''(z) + \kappa_0 x(z) = 0 \rightarrow \frac{1}{f} = l\kappa_0$$

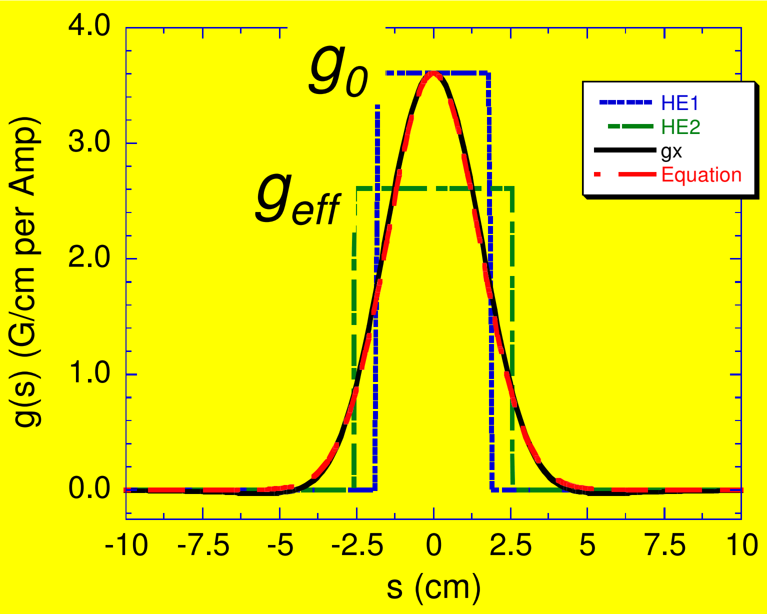


Smooth-Profile:

$$x''(z) + \kappa(z) x(z) = 0 \rightarrow \frac{1}{f} = l_{eff} \kappa_{peak}$$

$$l_{eff} = \frac{1}{\kappa_{peak}} \int_{-z_1}^{z_1} \kappa(z) dz$$

Effective Length of UMER Quadrupole*



Red curve is analytical fit to Mag-Li profile (black curve):

$$g(s)=g_0 exp(-s^2/d^2),$$

g_0=3.61 G/cmA, d = 2.10 cm.

Standard definition of effective length (which uses hardtop gradient g_0) yields

$$l_{eff} = 3.72 \text{ cm}$$

However, the same focal length can be obtained with a wider hardedge model with smaller hardtop gradient g_{eff}.

For the short UMER quad the correct hardedge model yields

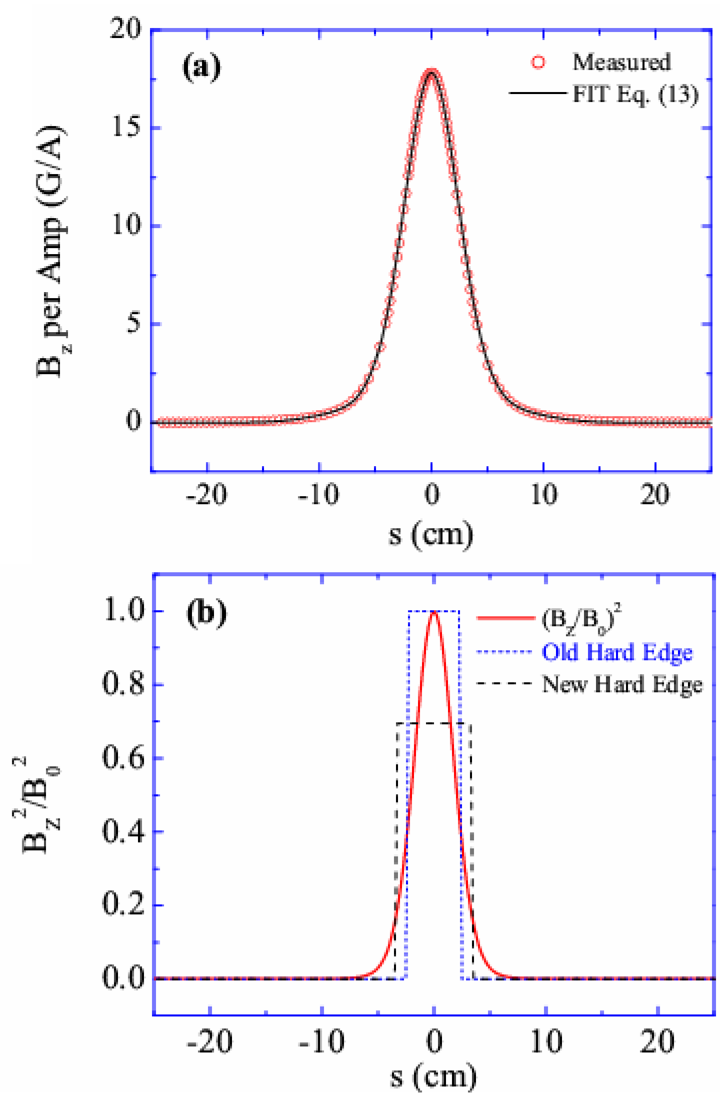
$$l_{eff} = 5.16 \text{ cm}, \; g_{eff} = 0.72\times g_0$$

*S. Bernal, et al, Phys, Rev. ST Accel. Beams, 9, 064202 (2006).

Effective Length of Short Solenoid*

UMER Solenoid Profile

$$B_Z(0,z) = B_0 \exp\left(-z^2/d^2\right)\left[\mathrm{sech}\left(z/b\right) + C_0 \sinh^2\left(z/b\right)\right]$$



Effective length calculated the standard way is $l_{eff} = 4.50$ cm

Similar issues as with the UMER quad…

For UMER short solenoid, new treatment yields

$l_{eff}(\mathrm{cm}) = 6.571\ \mathrm{cm} - 0.00029 \times \kappa_{peak}(\mathrm{m}^{-2})$,
$\kappa_{\mathrm{eff}}(\mathrm{m}^{-2}) = 0.6945 \times \kappa_{\mathrm{peak}}(\mathrm{m}^{-2})$

The effective length has a slight dependence on peak focusing function.

*S. Bernal, et al, Phys, Rev. ST Accel. Beams, 9, 064202 (2006).

References

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